Correction de l'interno 1

exo 1

(I) $u_0 = 0^2 + 0 = 0$ vraie.

(H) Sup qu'il existe un $n \in \mathbb{N}$ tq $u_n = n^2 + n$.

Mq $u_{n+1} = (n+1)^2 + n + 1 = n^2 + 2n + 1 + n + 1$
$= n^2 + 3n + 2$

On a $u_n = n^2 + n$

Dc $u_{n+1} = n^2 + n + 2n + 2$

$u_{n+1} = n^2 + 3n + 2$ vrai pour $n+1$.

(C) $\forall n \in \mathbb{N}$ $u_n = n^2 + n$

---

exo 1

1 a. $u_1 = \frac{3}{4}$ $u_2 = \frac{9}{10}$

b Mq $u_n > 0$

(I) $u_0 = \frac{1}{2} > 0$ vrai

(H) Sup $\exists n \in \mathbb{N}$ tq $u_n > 0$

Mq $u_{n+1} > 0$

On a $u_n > 0$ Dnc $3u_n > 0$ et $1 + 2u_n > 0$ $\Big\}$ Dnc $u_{n+1} > 0$

(C) $\forall n \in \mathbb{N}$, $u_n > 0$

2a. $u_{n+1} - u_n = \dfrac{3u_n}{1+2u_n} - u_n$

$= \dfrac{3u_n - u_n(1+2u_n)}{1+2u_n}$

$= \dfrac{u_n(3-1-2u_n)}{1+2u_n}$

$= \dfrac{u_n(2-2u_n)}{1+2u_n}$

$= \dfrac{2u_n(1-u_n)}{1+2u_n}$

Comme $u_n > 0$ alors $1 + 2u_n > 0$

et $u_n < 1$ Dnc $1 - u_n > 0$

Dnc $u_{n+1} - u_n > 0$

Dnc $(u_n) \nearrow$

b. (u_n) est croissante et majorée par 1.

Dnc (u_n) converge

3. a $\dfrac{v_{n+1}}{v_n} = \dfrac{\frac{u_{n+1}}{1-u_n}}{\frac{u_n}{1-u_n}} = \dfrac{\dfrac{\frac{3u_n}{1+2u_n}}{1-\frac{3u_n}{1+2u_n}}}{\frac{u_n}{1-u_n}} = \dfrac{\dfrac{\frac{3u_n}{1+2u_n}}{\frac{1+2u_n-3u_n}{1+2u_n}}}{\frac{u_n}{1-u_n}}$

$= \dfrac{\frac{3u_n}{1-u_n}}{\frac{u_n}{1-u_n}} = \dfrac{3u_n}{1-u_n} \times \dfrac{1-u_n}{3u_n} = 3.$ Dnc (v_n) géo de raison $q = 3$

b. $v_n = v_0 \cdot q^n$

$q = 3$

$v_0 = \frac{u_0}{1-u_0} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1.$

Donc $v_n = 3^n$

c. $v_n = \frac{u_n}{1-u_n}$.

Donc $(1-u_n)\, v_n = u_n$

$v_n - u_n \cdot v_n = u_n$

$v_n = u_n + u_n \cdot v_n$

$v_n = u_n\,(1+v_n)$

Donc $u_n = \frac{v_n}{1+v_n}$

$u_n = \frac{3^n}{1+3^n}$

d. $u_n = \frac{3^n}{3^n+1} = \frac{3^n}{3^n\left(1+\frac{1}{3^n}\right)} = \frac{1}{1+\frac{1}{3^n}}$

Comme $\lim\limits_{n\to+\infty} \frac{1}{3^n} = 0$ alors $\lim\limits_{n\to+\infty} u_n = 1$

exo3.

1. $\frac{8n^2-2n}{1-2n^2} = \frac{n^2\left(8-\frac{2}{n}\right)}{n^2\left(\frac{1}{n^2}-2\right)} = \frac{8-\frac{2}{n}}{\frac{1}{n^2}-2}$

or $\lim\limits_{n\to+\infty} \frac{2}{n} = \lim\limits_{n\to+\infty} \frac{1}{n^2} = 0$

Donc $\lim\limits_{n\to+\infty} \frac{8n^2-2n}{1-2n^2} = \frac{8}{-2} = -4$.

2. $-1 \leq \cos n \leq 1$

$-\frac{1}{n} \leq \frac{\cos n}{n} \leq \frac{1}{n}$ car $n > 0$

$\lim\limits_{n\to+\infty} -\frac{1}{n} = \lim\limits_{n\to+\infty} \frac{1}{n} = 0$

Donc $\lim\limits_{n\to+\infty} \frac{\cos n}{n} = 0$